TRINITY COLLEGE

IN NOMINE DOMINI

Semester Two Examination, 2022

Question/Answer booklet

MATHEMATICS SPECIALIST UNITS 1&2

Section Two: Calculator-assumed

SOLUTIONS

WA student number: In figures

In words

Your name

Number of additional answer booklets used (if applicable):

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	49	35
Section Two: Calculator-assumed	12	12	100	94	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed **65% (94 Marks)**

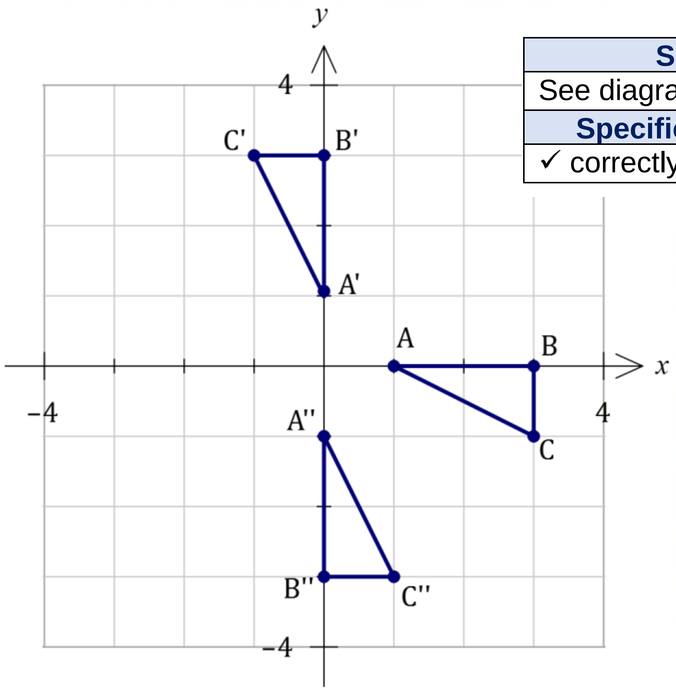
This section has **twelve** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 8 **(9 marks)**

Triangle ABC has vertices $A(1, 0)$, $B(3, 0)$ and $C(3, -1)$.

(a) Draw triangle ABC on the axes below, labelling all vertices. (1 mark)



Solution
See diagram
Specific behaviours
✓ correctly draws triangle

Triangle ABC is transformed by matrix $\mathbf{T}_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ to form triangle $A'B'C'$.

(b) Determine the coordinates of A', B' and C' and draw labelled triangle $A'B'C'$ on the axis above. (2 marks)

Solution
$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 3 & 3 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 \\ 1 & 3 & 3 \end{bmatrix}$ $A'(0, 1)$, $B'(0, 3)$, $C'(-1, 3)$
Specific behaviours
✓ correct coordinates   ✓ correctly draws transformed triangle

(c) Describe the geometric transformation that matrix $\mathbf{T}_1$ represents. (2 marks)

Solution
A reflection in the line $y = x$.
Specific behaviours
✓ states reflection   ✓ states line of reflection

Triangle $A'B'C'$ is rotated $180°$ about the origin to form triangle $A''B''C''$.

(d) Determine the transformation matrix $\mathbf{T}_2$ that represents the transformation from triangle $A'B'C'$ to triangle $A''B''C''$ and draw labelled triangle $A''B''C''$ on the same axis as triangle ABC. (2 marks)

Solution
$\mathbf{T}_2 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$
Specific behaviours
✓ correct matrix ✓ correctly draws transformed triangle

(e) Determine the single matrix $\mathbf{T}_3$ that will transform triangle $A''B''C''$ to triangle ABC and describe the geometric transformation that $\mathbf{T}_3$ represents. (2 marks)

Solution
$\mathbf{T}_3 = (\mathbf{T}_2\mathbf{T}_1)^{-1}$ $= \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$ $\mathbf{T}_3$ represents a reflection in the line $y = -x$.
Specific behaviours
✓ correct matrix ✓ correctly describes associated transformation

Question 9 **(8 marks)**

(a) Determine the number of integers between 1 and 2022 inclusive that are divisible by 17 or divisible by 5. (2 marks)

Solution
$\lfloor 2022 \div 17 \rfloor = 118$ $\lfloor 2022 \div 5 \rfloor = 404$ $\lfloor 2022 \div 85 \rfloor = 23$ $n = 118 + 404 - 23 = 499$
Specific behaviours
✓ indicates correct method ✓ correct answer

(b) Codes such as ZSTZT and XZUTY are made by randomly selecting five letters from the last eight letters in the alphabet. Determine what fraction of all possible codes contain repeated letters. (3 marks)

Solution
Codes with no restrictions, $n = 8^5 = 32\,768$. Codes with no repeated letters, $m = {}^8P_5 = 6720$. Hence fraction containing repeats is $\dfrac{32\,768 - 6720}{32\,768} = \dfrac{26\,048}{32\,768} = \dfrac{407}{512}$.
Specific behaviours
✓ total number of codes ✓ number of codes with no repeated letters ✓ correct fraction

(c) Determine the number of different ways that the digits in the number 38723945 can be arranged so that all the odd digits are next to each other. (3 marks)

Solution
Group odd digits together in $\dfrac{5!}{2!} = 60$ ways. Arrange 3 even digits and group in $4! = 24$ ways. Hence number of different ways is $60 \times 24 = 1440$.
Specific behaviours
✓ groups odd digits together ✓ arranges ✓ correct number of ways

Question 10 **(8 marks)**

(a) Express a displacement of 75 m on a bearing of $311°$ in component form to one decimal place, given that the unit vectors $\mathbf{i}$ and $\mathbf{j}$ are directed due east and north respectively. (2 marks)

Solution
Angle from positive x-axis is $90° - 311° + 360° = 139°$. $75\begin{pmatrix}\cos(139°)\\ \sin(139°)\end{pmatrix} = \begin{pmatrix}-56.6\\ 49.2\end{pmatrix}$
Specific behaviours
✓ indicates appropriate method   ✓ correct components

(b) Express the velocity $\begin{pmatrix}97\\ -78\end{pmatrix}$ m/s as a linear combination of the velocities $\begin{pmatrix}5\\ -2\end{pmatrix}$ m/s and $\begin{pmatrix}-3\\ 4\end{pmatrix}$ m/s. (3 marks)

Solution
$\begin{pmatrix}97\\ -78\end{pmatrix} = x\begin{pmatrix}5\\ -2\end{pmatrix} + y\begin{pmatrix}-3\\ 4\end{pmatrix}$ $5x - 3y = 97$ $-2x + 4y = -78$ $x = 11, \quad y = -14$ $\begin{pmatrix}97\\ -78\end{pmatrix} = 11\begin{pmatrix}5\\ -2\end{pmatrix} - 14\begin{pmatrix}-3\\ 4\end{pmatrix}$
Specific behaviours
✓ forms vector equation   ✓ writes simultaneous equations   ✓ solves equations for x and y and writes as required

(c) In the trapezium $OXYZ$, $\overrightarrow{OX} = \mathbf{x}$, $\overrightarrow{OZ} = \mathbf{z}$, $\overrightarrow{ZY} = 3\mathbf{x}$ and M and N are the midpoints of XY and YZ respectively. Determine the value of the constant μ and the value of the constant λ if $\overrightarrow{MN} = \mu\mathbf{x} + \lambda\mathbf{z}$. (3 marks)

Solution
$\overrightarrow{ON} = \overrightarrow{OZ} + \frac{1}{2}\overrightarrow{ZY} = \mathbf{z} + \frac{1}{2}(3\mathbf{x}) = \mathbf{z} + \frac{3}{2}\mathbf{x}$ $\overrightarrow{XY} = \overrightarrow{XO} + \overrightarrow{OZ} + \overrightarrow{ZY} = -\mathbf{x} + \mathbf{z} + 3\mathbf{x} = \mathbf{z} + 2\mathbf{x}$ $\overrightarrow{OM} = \overrightarrow{OX} + \frac{1}{2}\overrightarrow{XY} = \mathbf{x} + \frac{1}{2}(\mathbf{z} + 2\mathbf{x}) = \frac{1}{2}\mathbf{z} + 2\mathbf{x}$ $\overrightarrow{MN} = \overrightarrow{ON} - \overrightarrow{OM} = \left(\mathbf{z} + \frac{3}{2}\mathbf{x}\right) - \left(\frac{1}{2}\mathbf{z} + 2\mathbf{x}\right) = \frac{1}{2}\mathbf{z} - \frac{1}{2}\mathbf{x}$ Hence $\mu = -\frac{1}{2}$ and $\lambda = \frac{1}{2}$.
Specific behaviours
✓ vector for $\overrightarrow{ON}$ ✓ vector for $\overrightarrow{OM}$ ✓ vector for $\overrightarrow{MN}$, stating value of μ and value of λ

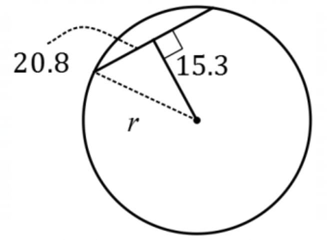
Question 11 **(8 marks)**

(a) A chord of length 20.8 cm is 15.3 cm away from the centre of a circle. Determine the distance of a chord of length 35.2 cm from the centre of the same circle. (3 marks)

Solution

$$r^2 = 15.3^2 + \left(\frac{20.8}{2}\right)^2 = d^2 + \left(\frac{35.2}{2}\right)^2$$

$$d = 5.7 \text{ cm}$$



Specific behaviours

✓ sketch showing right-triangle and lengths
✓ indicates expression for circle radius
✓ correct distance of chord

(b) Chords PR and QS of the circle with centre O are perpendicular to each other and intersect at T. Determine, with reasoning, the distance OT when $RT = 7$ cm, $ST = 14$ cm and $QS = 16$ cm. (5 marks)

Solution

Using intersecting chord theorem:

$$PT \times RT = ST \times QT$$

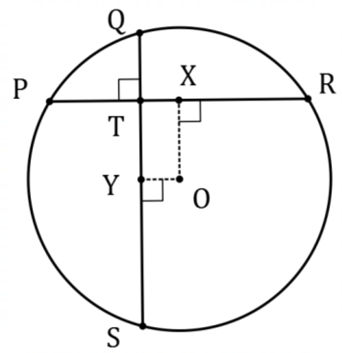
$$PT = \frac{14 \times (16 - 14)}{7} = 4$$

Let X and Y be the midpoints of PR and QS respectively so that $OX \perp PR$ and $OY \perp QS$.

Then $TX = PX - PT = \dfrac{11}{2} - 4 = 1.5$

and $TY = QY - QT = \dfrac{16}{2} - 2 = 6.$

Hence $OT = \sqrt{1.5^2 + 6^2} = \dfrac{3\sqrt{17}}{2} \approx 6.18$ cm.



Specific behaviours

✓ labelled sketch that shows intersecting chords
✓ uses intersecting chord theorem for PT
✓ uses midpoints to form rectangle $OXTY$
✓ dimensions of rectangle
✓ distance OT

Question 12 **(7 marks)**

A small body is oscillating vertically on a spring in a laboratory so that the height h cm of its centre above the floor t seconds after measurements began can be modelled by

$$h = a\cos\big(b(t-c)\big) + 75.$$

The maximum height of the body of 130 cm is first reached when $t = 1$ and again when $t = 9$.

(a) Determine the value of each of the positive constants a, b and c. If more than one value is possible, give the value closest to 0. (3 marks)

Solution
When $\cos\big(b(t-c)\big) = 1$ then $130 = a + 75 \to a = 55$.
Period of motion is $9 - 1 = 8 \to b = 2\pi \div 8 = \frac{\pi}{4}$.
Maximum when $t = 1 \to \cos\left(\frac{\pi}{4}(1-c)\right) = 1 \to c = 1$.
$a = 55, \quad b = \frac{\pi}{4}, \quad c = 1$
Specific behaviours
✓ value of a ✓ value of b ✓ value of c

The ceiling is 2.2 m above the floor.

(b) If d cm is the distance from the centre of the body to the ceiling, determine the relationship between d and t as a sine function. (2 marks)

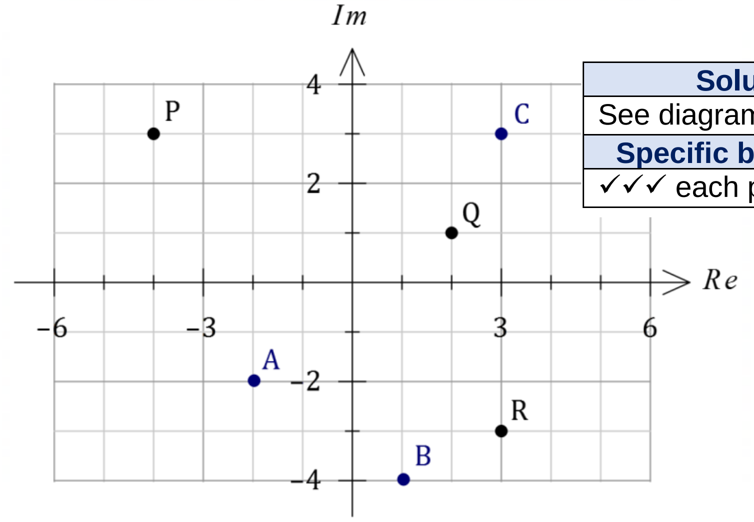
Solution
Since $\cos A = \sin\left(A + \frac{\pi}{2}\right)$ then $h = 55\sin\left(\frac{\pi}{4}(t-1) + \frac{\pi}{2}\right) + 75$.
$d = 220 - h$ $= 220 - \left(55\sin\left(\frac{\pi}{4}(t-1) + \frac{\pi}{2}\right) + 75\right)$ $= 145 - 55\sin\left(\frac{\pi t}{4} + \frac{\pi}{4}\right)$
Specific behaviours
✓ converts h to sine function ✓ correct model for d

(c) At what time is the centre of the body first equidistant from the ground and the ceiling? (2 marks)

Solution
$145 - 55\sin\left(\frac{\pi t}{4} + \frac{\pi}{4}\right) = \frac{220}{2} \to t = 2.12$ s
Specific behaviours
✓ indicates correct equation ✓ correct time

Question 13 **(8 marks)**

The points P, Q and R shown in the complex plane below represent the complex numbers z_1, z_2 and z_3 respectively.



Solution
See diagram
Specific behaviours
✓✓✓ each point

(a) On the same diagram plot the points A, B and C to represent the following complex numbers. (3 marks)

$$A: z_1 + 2 - 5i, \qquad B: z_3 - z_2, \qquad C: \overline{z_3}.$$

(b) Determine

(i) $\text{Re}((z_2)^2)$. (1 mark)

(ii) $\text{Im}\left(z_3 \div \left(1 - \sqrt{2}i\right)\right)$. (1 mark)

Solution
$z_2 = 2 + i \rightarrow \text{Re}((2+i)^2) = 3$
$z_3 = 3 - 3i \rightarrow \text{Im}\left(\dfrac{3-3i}{1-\sqrt{2}i}\right) = \sqrt{2} - 1$
NB Use of CAS is efficient here for 1 mark.
Specific behaviours
✓ correct answer to (i)
✓ correct answer to (ii)

(c) Given that the complex number z_1 is a solution to the equation $z^2 + bz + c = 0$, determine the value of the real constant b and the real constant c. (3 marks)

Solution
$z_1 = -4 + 3i, \quad \overline{z_1} = -4 - 3i$
$z^2 + bz + c = \big(z - (-4+3i)\big)\big(z - (-4-3i)\big)$ $= z^2 + 8z + 25$
$b = 8, \quad c = 25$
Specific behaviours
✓ indicates conjugate is also a root
✓ forms factored expression
✓ expands and states both values

Alternative Solution
$z_1 = -4 + 3i, \quad \overline{z_1} = -4 - 3i$
$b = -\big((-4+3i) + (-4-3i)\big) = 8$
$c = (-4+3i) \times (-4-3i) = 25$
Specific behaviours
✓ indicates conjugate is also a root
✓ shows use of sum and product of roots
✓ states both values

Question 14 **(7 marks)**

(a) Prove that $\sin(\theta + \phi) - \sin(\theta - \phi) = 2\cos\theta\sin\phi$. (2 marks)

Solution
$LHS = \sin(\theta + \phi) - \sin(\theta - \phi)$ $= (\sin\theta\cos\phi + \cos\theta\sin\phi) - (\sin\theta\cos\phi - \cos\theta\sin\phi)$ $= 2\cos\theta\sin\phi$ $= RHS$
Specific behaviours
✓ uses sum and difference identities ✓ shows logical steps and simplifies to complete proof

(b) Express the difference $\sin 7x - \sin x$ as a product in the form $k\sin\alpha\cos\beta$. (1 mark)

Solution
$\sin(4x + 3x) - \sin(4x - 3x) = 2\cos 4x \sin 3x$
Specific behaviours
✓ correctly uses difference to product identity

(c) Determine all exact solutions of the equation $\sin 7x - \cos 4x - \sin x = 0$ for $0 \le x \le \frac{\pi}{2}$, justifying your answer. (4 marks)

Solution
$\sin 7x - \sin x - \cos 4x = 0$ $2\cos 4x\sin 3x - \cos 4x = 0$ $\cos 4x\,(2\sin 3x - 1) = 0$ $\cos 4x = 0$ $4x = \frac{\pi}{2}, \frac{3\pi}{2}$ $x = \frac{\pi}{8}, \frac{3\pi}{8}$ $2\sin 3x - 1 = 0$ $\sin 3x = \frac{1}{2}$ $3x = \frac{\pi}{6}, \frac{5\pi}{6}$ $x = \frac{\pi}{18}, \frac{5\pi}{18}$ $x = \frac{\pi}{18}, \frac{\pi}{8}, \frac{5\pi}{18}, \frac{3\pi}{8}$
Specific behaviours
✓ uses product from (a) ✓ obtains linear factored form of equation ✓ solutions from one factor ✓ solutions from second factor

Question 15 **(8 marks)**

(a) Two forces are given by $\mathbf{F}_1 = \begin{pmatrix} 33 \\ 16.5 \end{pmatrix}$ N and $\mathbf{F}_2 = \begin{pmatrix} -12 \\ 9 \end{pmatrix}$ N. Determine

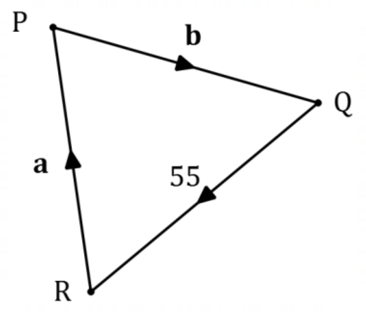
(i) the component of $\mathbf{F}_1$ that is acting parallel to $\mathbf{F}_2$. (2 marks)

Solution
Require vector projection of $\mathbf{F}_1$ on $\mathbf{F}_2$: $$\left(\frac{\mathbf{F}_1 \cdot \mathbf{F}_2}{\mathbf{F}_2 \cdot \mathbf{F}_2}\right)\mathbf{F}_2 = \left(\frac{\begin{pmatrix} 33 \\ 16.5 \end{pmatrix} \cdot \begin{pmatrix} -12 \\ 9 \end{pmatrix}}{\begin{pmatrix} -12 \\ 9 \end{pmatrix} \cdot \begin{pmatrix} -12 \\ 9 \end{pmatrix}}\right)\begin{pmatrix} -12 \\ 9 \end{pmatrix} = -1.1\begin{pmatrix} -12 \\ 9 \end{pmatrix} = \begin{pmatrix} 13.2 \\ -9.9 \end{pmatrix} \text{ N}$$
Specific behaviours
✓ indicates correct method ✓ correct component

(ii) the component of $\mathbf{F}_1$ that is acting perpendicular to $\mathbf{F}_2$. (2 marks)

Solution
If $\mathbf{F}$ is the required component, then $\mathbf{F} + \begin{pmatrix} 13.2 \\ -9.9 \end{pmatrix} = \mathbf{F}_1$ and so $$\mathbf{F} = \begin{pmatrix} 33 \\ 16.5 \end{pmatrix} - \begin{pmatrix} 13.2 \\ -9.9 \end{pmatrix} = \begin{pmatrix} 19.8 \\ 26.4 \end{pmatrix} \text{ N}$$
Specific behaviours
✓ indicates correct method ✓ correct component

(b) A small body, acted on by three horizontal forces, is in equilibrium on a smooth level table. One force has magnitude 55 N and acts on a bearing of $245°$. Determine the magnitudes of the other two forces given that they act on bearings of $105°$ and $350°$. (4 marks)

Solution	
$\angle P = 180° - 105° - 10° = 65°$ $\angle Q = 180° - 75° - 65° = 40°$ $\angle R = 180° - 65° - 40° = 75°$ Hence $\dfrac{\lvert\mathbf{a}\rvert}{\sin 40°} = \dfrac{55}{\sin 65°} \Rightarrow \lvert\mathbf{a}\rvert = 39.0 \text{ N}$ and $\dfrac{\lvert\mathbf{b}\rvert}{\sin 75°} = \dfrac{55}{\sin 65°} \Rightarrow \lvert\mathbf{b}\rvert = 58.6 \text{ N}$	
Specific behaviours	
✓ sketch diagram ✓ calculates all angles in triangle ✓ one correct magnitude ✓ second correct magnitude	

Question 16 **(6 marks)**

(a) Balls are taken at random and without replacement from a bag that contains 6 pink, 8 yellow, 11 green, 12 silver, 13 blue and 15 red balls. Demonstrate use of the pigeon-hole principle to determine the least number of balls that should be taken from the bag to be certain that at least 10 of them are of the same colour. (3 marks)

Solution
There are 65 pigeons (balls) and 6 pigeon-holes (ball colours). Fill pigeon-holes as follows: pink and yellow with $6 + 8 = 14$ balls and green, silver, blue and red with $9 + 9 + 9 + 9 = 36$ balls. Now there are only green, silver, blue and red balls left in the bag and so picking one more will ensure that we have at least 10 of the same colour. Hence, we must take $14 + 36 + 1 = 51$ balls from the bag.
Specific behaviours
✓ indicates balls are pigeons and colours are pigeon-holes ✓ correctly fills pigeon-holes ✓ correct number of balls

(b) A school social committee of 4 people is to be selected from a group of 11 secondary students, 7 primary students and 12 teachers. Determine how many of the possible committees have more primary students than teachers. (3 marks)

Solution
All primary: $\binom{7}{4} = 35$. Three primary and one other: $\binom{7}{3}\binom{23}{1} = 35 \times 23 = 805$. Two primary, one teacher, one secondary: $\binom{7}{2}\binom{12}{1}\binom{11}{1} = 21 \times 12 \times 11 = 2772$. Two primary, two secondary: $\binom{7}{2}\binom{11}{2} = 21 \times 55 = 1155$. One primary, three secondary: $\binom{7}{1}\binom{11}{3} = 7 \times 165 = 1155$. Total number of committees is $35 + 805 + 2772 + 1155 + 1155 = 5922$.
Specific behaviours
✓ identifies at least four possible cases ✓ calculates at least four cases correctly ✓ correct total number of committees

Question 17 **(8 marks)**

A small plane that has a cruising speed of 221 km/h is planning to fly from airport A to airport B, where $\overrightarrow{AB} = \begin{pmatrix} -429 \\ 363 \end{pmatrix}$ km and a steady wind of $\begin{pmatrix} 15 \\ -8 \end{pmatrix}$ km/h is forecast for the local region.

Let the velocity vector that the small plane should set for the flight be $\begin{pmatrix} p \\ q \end{pmatrix}$ km/h.

(a) Explain why

(i) $\sqrt{p^2 + q^2} = 221.$ (1 mark)

Solution
Magnitude of velocity vector must equal the cruising speed.
Specific behaviours
✓ correct explanation

(ii) $p + 15 = -429k$ and $q - 8 = 363k$, where k is a constant. (2 marks)

Solution
The resultant velocity must be parallel to $\overrightarrow{AB}$ and so $\begin{pmatrix} p+15 \\ q-8 \end{pmatrix} = k\begin{pmatrix} -429 \\ 363 \end{pmatrix}$. By equating $\mathbf{i}$ and $\mathbf{j}$-coefficients we obtain $p + 15 = -429k$ and $q - 8 = 363k$.
Specific behaviours
✓ indicates resultant must be parallel to $\overrightarrow{AB}$ ✓ equates $\mathbf{i}$ and $\mathbf{j}$-coefficients

(b) Determine another relationship between p and q, besides $\sqrt{p^2 + q^2} = 221$. (2 marks)

Solution
$k = \dfrac{p+15}{-429} = \dfrac{q-8}{363}$
Specific behaviours
✓ solves $\mathbf{i}$-coefficient equation for k ✓ solves $\mathbf{j}$-coefficient equation for k and equates to eliminate k

(c) Determine the velocity vector $\begin{pmatrix} p \\ q \end{pmatrix}$ and hence calculate the expected flight time for the small plane from A to B. (3 marks)

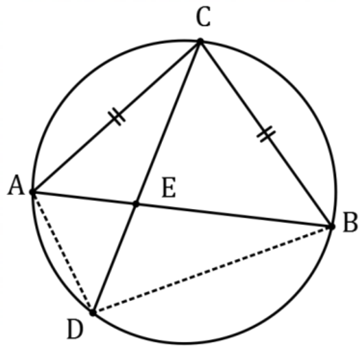
Solution
Solve the following equations simultaneously using CAS: $\dfrac{p+15}{-429} = \dfrac{q-8}{363}, \qquad \sqrt{p^2+q^2} = 221$ Two solution sets arise, but the vector in required direction is $\begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} -171 \\ 140 \end{pmatrix}$. Using $\mathbf{j}$-coefficient, $t(140 - 8) = 363 \Rightarrow t = 2.75$ hours.
Specific behaviours
✓ indicates method ✓ obtains correct velocity vector ✓ correct flight time

Question 18 **(9 marks)**

Points A, B and C lie on a circle and join to form an isosceles triangle in which $CA = CB$. Let E be a point on the base AB so that CE extended meets the circle at D.

(a) Sketch a diagram to show the above information. (1 mark)

Solution



Specific behaviours

✓ correct labelled diagram (dotted lines optional)

(b) Use angle theorems to prove that triangle AED is similar to triangle CEB. (3 marks)

Solution

$\angle AED = \angle CEB$ (opposite angles)

$\angle ADC = \angle ABC$ (angles on same arc)

Hence $\Delta AED \sim \Delta CEB$ (AAA)

Specific behaviours

✓ shows one pair of angles equal with reasoning
✓ shows second pair of angles equal with reasoning
✓ deduces similarity with reasoning

(c) Show that $AD \times BE = DE \times CB$. (1 mark)

Solution

Since $\Delta AED \sim \Delta CEB$ then the ratio of corresponding sides will be equal:

$$\frac{AD}{CB} = \frac{DE}{BE} \Rightarrow AD \times BE = DE \times CB$$

Specific behaviours

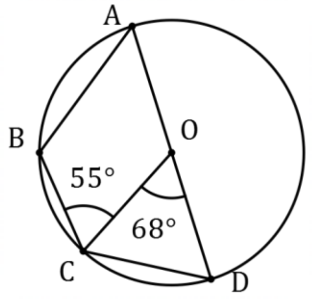
✓ shows ratio of corresponding sides

(d) Prove that $\frac{1}{AD}+\frac{1}{BD}=\frac{1}{DE}\times\frac{AB}{CB}$. (4 marks)

Solution
To relate BD to ED, first prove that $\Delta BED \sim \Delta CEA$: $\angle AEC = \angle DEB$ (opposite angles) $\angle BDC = \angle BAC$ (angles on same arc) Hence $\Delta BED \sim \Delta CEA$ (AAA) and so $BD \times AE = DE \times CA$. Proof: $LHS = \frac{1}{AD}+\frac{1}{BD}$ $= 1 \div \left(\frac{DE \times CB}{BE}\right) + 1 \div \left(\frac{DE \times CA}{AE}\right)$ (Using ratio of sides) $= \frac{BE}{DE \times CB} + \frac{AE}{DE \times AC}$ $= \frac{BE}{DE \times CB} + \frac{AE}{DE \times CB}$ (ΔABC is isosceles so $CA = CB$) $= \frac{BE + AE}{DE \times CB}$ $= \frac{AB}{DE \times CB}$ ($AE + EB = AB$) $= \frac{1}{ED} \times \frac{AB}{CB}$ $= RHS$
Specific behaviours
✓ proves that $\Delta BED \sim \Delta CEA$ ✓ substitutes for AD and BD ✓ eliminates AC with reasoning and combines fractions ✓ completes proof

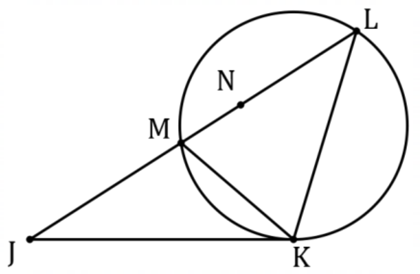
Question 19 **(8 marks)**

(a) The points A, B, C and D lie in order on a semi-circle with centre O and diameter AOD. When $\angle DOC = 68°$ and $\angle BCO = 55°$, determine the size of $\angle OAB$. (3 marks)

Solution	
$\angle OCD = (180° - 68°) \div 2 = 56°$ $\angle OAB = 180° - (55° + 56°) = 69°$	
Specific behaviours	
✓ diagram, ✓ size of $\angle OCD$, ✓ correct angle	

(b) In the diagram, JK is the tangent to the circle at K and JML is a secant that cuts the circle at M and L.

Point N lies between M and L so that $JK = JN$.



(i) Prove that $\angle JMK = \angle JKL$. (2 marks)

Solution
$\angle JMK = \angle MKL + \angle MLK$ (exterior is sum of opposite interior angles) $= \angle MKL + \angle MKJ$ $(\angle MLK = \angle MKJ$ alternate segment angles) $= \angle JKL$
Specific behaviours
✓ uses exterior angle property of triangle ✓ uses alternate segment angle property

(ii) Prove that KN bisects $\angle MKL$. (3 marks)

Solution
$\angle JKN = \angle MKJ + \angle MKN, \quad \angle JNK = \angle NKL + \angle MLK$ But $\angle JKN = \angle JNK$ as ΔJKN is isosceles. Hence $\angle MKJ + \angle MKN = \angle NKL + \angle MLK$ $\angle MKJ + \angle MKN = \angle NKL + \angle MKJ$ $(\angle MLK = \angle MKJ$ alternate segments) $\angle MKN = \angle NKL$ Hence KN bisects $\angle MKL$.
Specific behaviours
✓ equates expressions for angles in isosceles triangle ✓ uses alternate angle property ✓ simplifies to show that KN is bisector

End of questions

Supplementary page

Question number: _________

Supplementary page

Question number: _________